Exam. Code : 103205
Subject Code : 9261

B.A./B.Sc. 5th Semester (Old Sylb. 2016)

MATHEMATICS

(Vector Calculus and Solid Geometry)

Paper—I

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions in all, choosing at least **TWO** from each section.

SECTION—A

I. (a) Define limit and continuity of a vector function. Derive the derivative of vector function $\vec{r} = \vec{f}(t)$ in terms of limit.

(b) If $\vec{r} \times d\vec{r} = 0$, show that $\vec{r}$ is a constant. 5,5

II. (a) Define curl of a vector point function and discuss its physical interpretation.

(b) Find the directional derivative of $f(x, y, z) = x^2y^3z^2$ at the point (1, 2, –1) in the direction of tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$.

(c) Prove that $\nabla \log|\vec{r}| = \frac{\vec{r}}{r^2}$. 4,4,2

III. (a) Prove that $r^n\vec{r}$ is irrotational. Find n when it is solenoidal.

(b) Find grad r^m, where r is the distance of any point from the origin. 5,5

IV. (a) State and prove Green's theorem in a plane.

(b) Prove that $\text{div curl}\,\vec{f} = 0$, where $\vec{f}$ in any continuously differentiable vector point function. 7,3

V. (a) Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0).

(b) State Stoke's theorem. 8,2

SECTION—B

VI. (a) Trace the locus of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$, where a, b, c are positive.

(b) Obtain the equation of the surface of revolution obtained by rotating the curve $y^2 + 9z^2 = 36$, $x = 0$ about the z-axis. 7,3

VII. (a) Find the equation of the tangent plane at the point (x_1, y_1, z_1) of the central conicoid $ax^2 + by^2 + cz^2 = 1$.

(b) A tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinate axes in A, B, C. Prove that the centroid of the triangle ABC lies on :

$$\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9.$$ 5,5

VIII. (a) Prove that there are six points on an ellipsoid the normals at which pass through a given point (l, m, n).

(b) Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2 + by^2 + cz^2 = 1$ at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2.$$ 4,6

IX. (a) Find the equation of the enveloping cone from the point (x_1, y_1, z_1) to the paraboloid $ax^2 + by^2 = 2cz$.

(b) Find the equation of the surface on which the normals from the point (α, β, γ) to the elliptic paraboloid $x^2 + 2y^2 = 4z$ lies. 4,6

X. (a) Show that if the origin is the centre of a conicoid, the coefficients of the first degree terms in its equation are all zero.

(b) Reduce the equation

$$11x^2 + 10y^2 + 6z^2 - 8yz + 4zx - 12xy + 72x - 72y + 36z + 150 = 0$$

to the standard form and show that it represents an ellipsoid and find the equations of the axes. 3,7